



**NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY**

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION:	Bachelor of Science Honours in Applied Mathematics		
QUALIFICATION CODE:	08BSMH	LEVEL:	8
COURSE CODE:	ACA801S	COURSE NAME:	ADVANCED COMPLEX ANALYSIS
SESSION:	JUNE 2022	PAPER:	THEORY
DURATION:	3 HOURS	MARKS:	100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSI NGUETCHUE
MODERATOR:	Prof F. MASSAMBA

INSTRUCTIONS
<ol style="list-style-type: none">1. Answer ALL the questions in the booklet provided.2. Show clearly all the steps used in the calculations3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1 [15 marks]

Let $O \subset \mathbb{C}$ be an open set and let $f: O \rightarrow \mathbb{C}$ be a holomorphic function.

1.1 What is an isolate singularity of f ? [3]

1.2 When is $c \in O$ a removable singularity and how does one remove such a singularity? [4]

1.3 What is an essential singularity of f ? [3]

1.4 What is a pole of f and what is the order of a pole? [5]

Problem 2 [30 marks]

2.1 Determine the order of the pole of each of the following functions at the indicated point:

2.1.1 $f(x) = \frac{1}{z \sin z}$ at $z_0 = 0$; [6]

2.1.2 $f(x) = \frac{e^{z^2} - 1}{z^4}$ at $z_0 = 0$; [6]

2.2 Show that the functions given by $f(x) = \frac{\sin z}{z}$ at $z_0 = 0$ and $g(x) = \frac{e^{z-1} - 1}{z - 1}$ at $z_0 = 1$ possess a removable singularity at the indicated point. [9]

2.3 For the given functions $f(z) = (z^2 - 1)\frac{1}{z - 1}$ and $g(x) = \frac{z^2}{(z - i)^3}$, determine whether they possess: [9]

- (i) Removable singularity;
- (ii) Pole(s), or
- (iii) Essential singularity.

If it is a pole, then determine the order of the pole.

Problem 3 [25 marks]

Let $\sum_{k=0}^{\infty} a_k(z - c)^k$ be a convergent power series and $\varepsilon > 0$ such that $B_\varepsilon(c) \subset D(c, R)$, where $D(c, R)$ is the disk of convergence of the power series.

Let $f: B_\varepsilon(c) \rightarrow \mathbb{C}$ be defined by

$$f(z) := \sum_{k=0}^{\infty} a_k(z - c)^k.$$

3.1 Prove that f is n -times differentiable for all $n \in \mathbb{N}$ and that

$$f^{(n)}(z) = \sum_{k=n}^{\infty} k(k-1)\cdots(k-n+1)a_k(z-c)^{k-n}$$

for all $n \in \mathbb{N}$ and all $x \in B_\varepsilon(c)$. With respect to differentiability what kind of function is f ? [15]

3.2 Show that

$$\frac{f^{(n)}(c)}{n!} = a_n, \quad \text{for all } n \in \mathbb{N}_0.$$

What does this mean for the power series? [7]

3.2 What is the Taylor series of f at c ? [3]

Problem 4 [30 marks]

4.1 State the Laurent series Theorem for a function of complex variable. [4]

4.2 Find the Laurent series of $f(z) = \frac{1}{1-z}$ for $1 < |z|$. [7]

4.3 Let $f: \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$ be defined by

$$f(z) := e^{-\frac{1}{z}}.$$

4.3.1 Find the Laurent series of f about $z_0 = 0$. [5]

4.3.2 What kind of singularity is $z_0 = 0$? How does f behave in the vicinity of $z_0 = 0$? [5]

4.3.3 State the residue Theorem. [4]

4.3.4 Find [5]

$$\int_{C_1(0)} e^{-\frac{1}{\zeta}} d\zeta$$

END